

# Chapter 1

## Right Triangle Trigonometry

1. **Angles, Degrees, and Triangles**
2. Similar Triangles
3. Right Triangle Ratios
4. Evaluating Trigonometric Functions
5. Solving Right Triangles



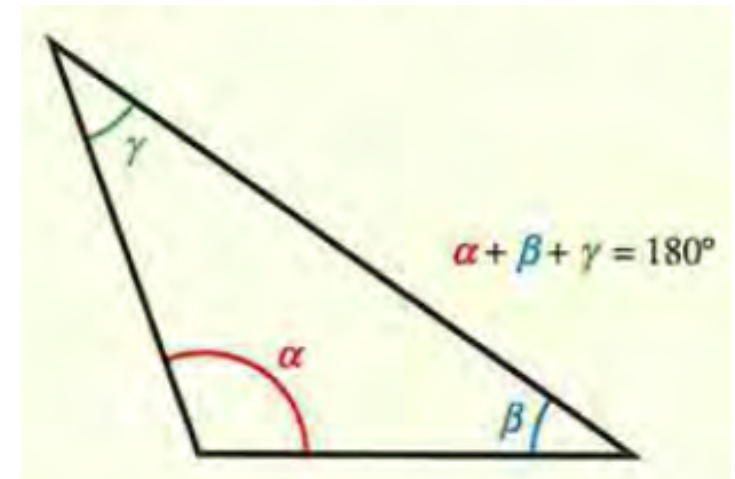
# Basics of Triangles

# 1.1 - Angles, Degrees, and Triangles

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## Triangle Sum Theorem

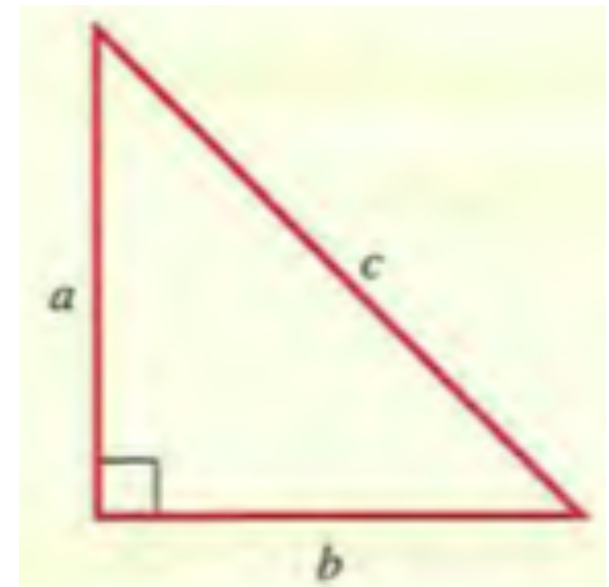
The sum of the interior angles is  $180^\circ$ .



## Pythagorean Theorem

In any right triangle

$$a^2 + b^2 = c^2$$



# 1.1 - Angles, Degrees, and Triangles

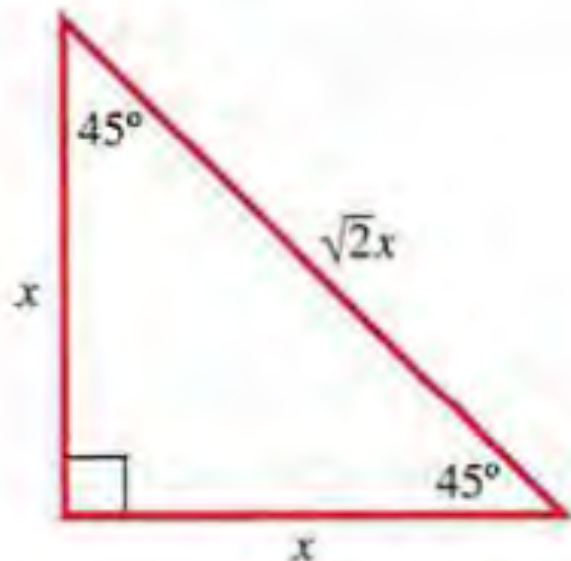
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## Special Right Triangles

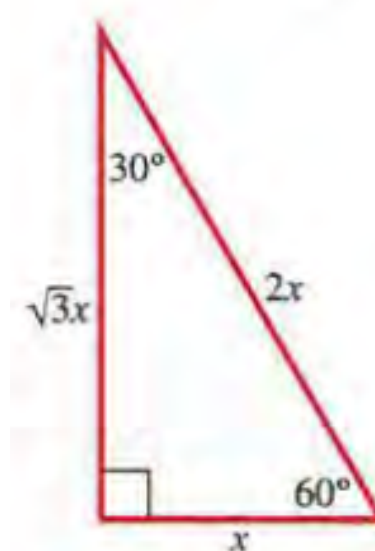
- **Pythagorean Triples** - Integer numbers that satisfy the Pythagorean Theorem.

$$3^2 + 4^2 = 5^2 \quad 5^2 + 12^2 = 13^2 \quad 8^2 + 15^2 = 17^2$$

- **45°-45°-90°**



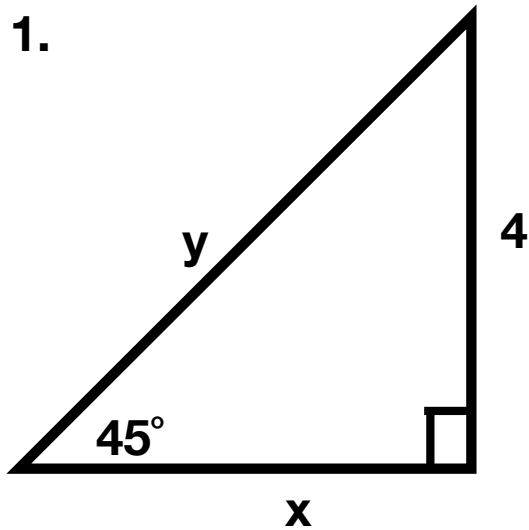
- **30°-60°-90°**



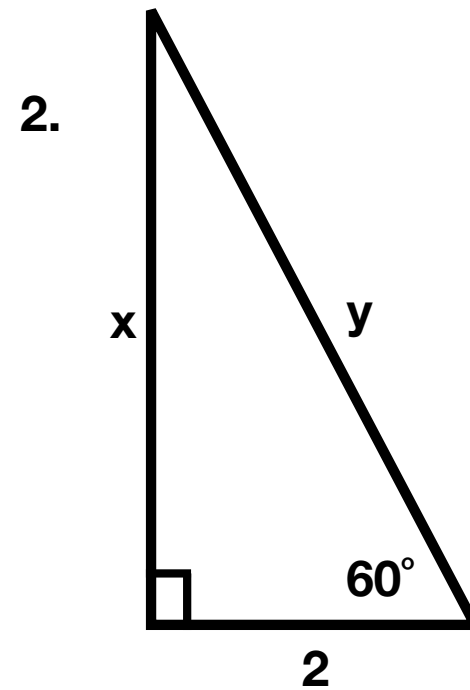
# 1.1 - Angles, Degrees, and Triangles

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## Practice



$$x = 4$$
$$y = 4\sqrt{2}$$



$$x = 2\sqrt{3}$$
$$y = 4$$

# 1.1 - Angles, Degrees, and Triangles

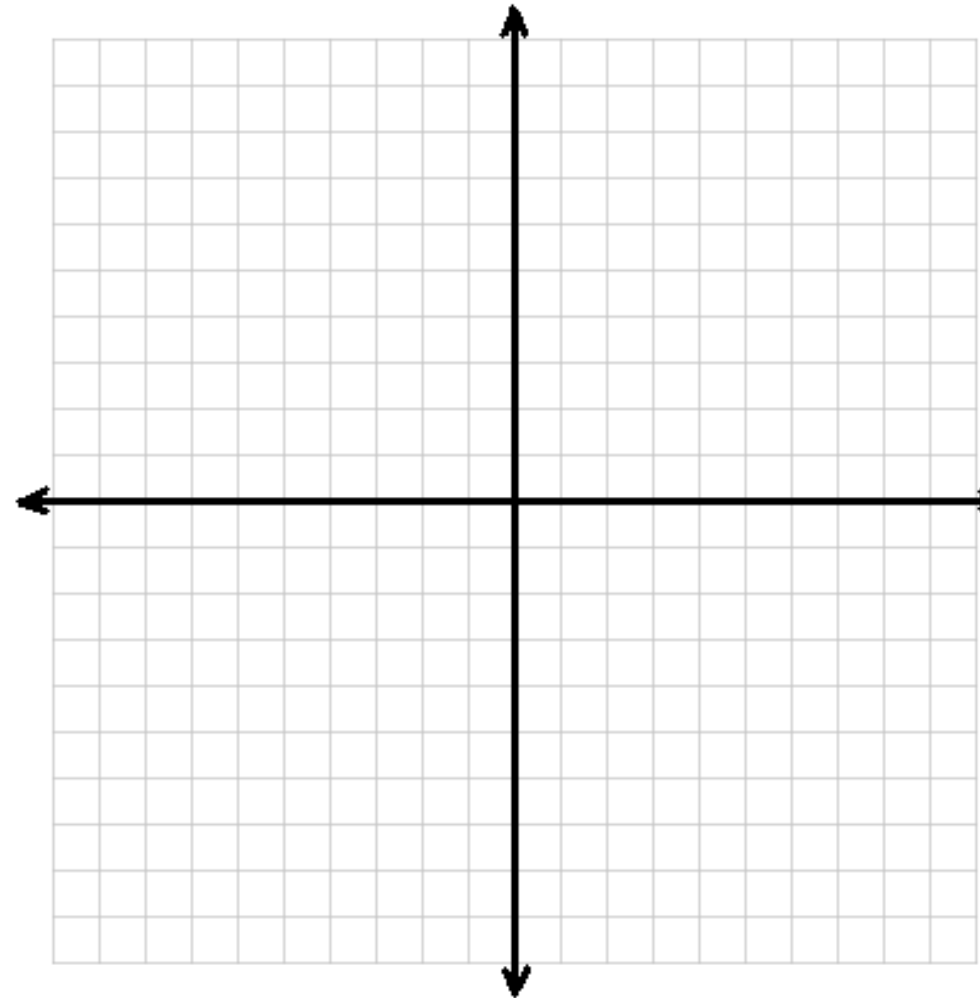
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## Real World Problem

### London Eye

The wheel takes 30 minutes to make a complete rotation. What will it rotate in 12 minutes?

**144°**



# 1.1 - Angles, Degrees, and Triangles

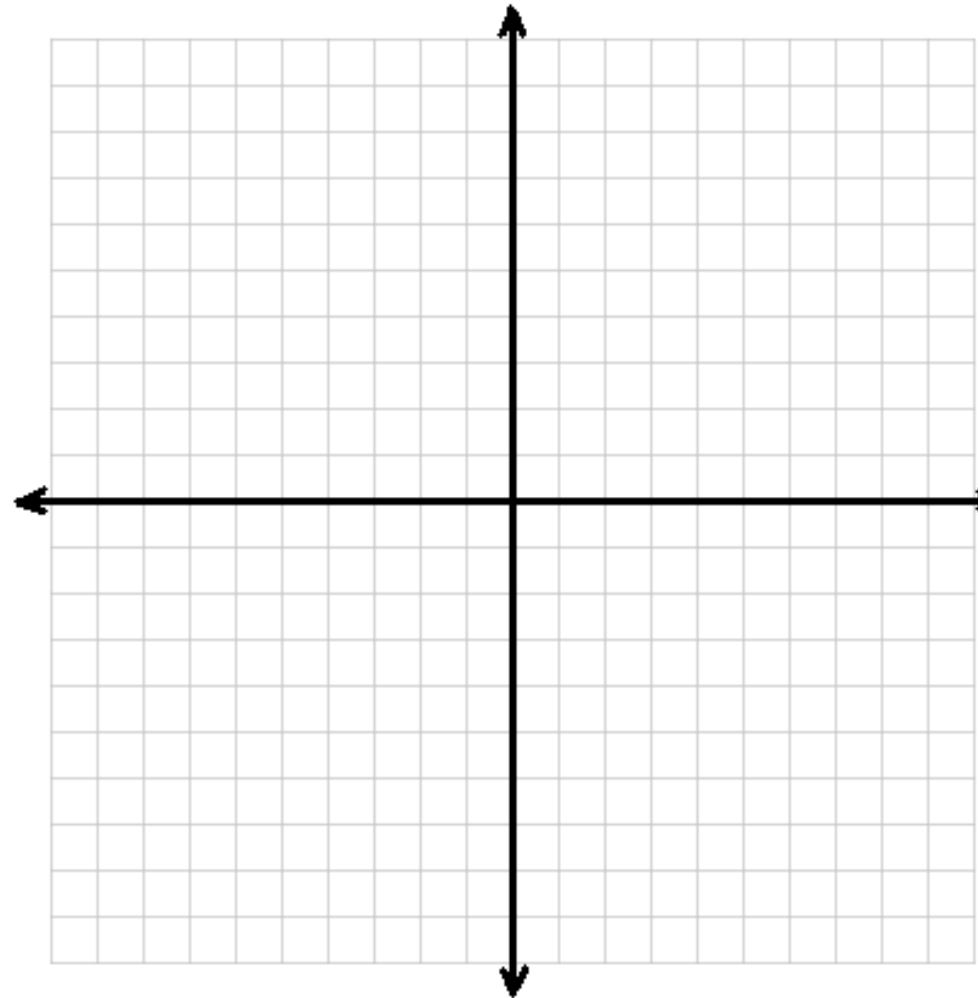
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## Real World Problem

### Clock

The hands of a clock are at 10:40. What acute angle do the big and little hands make?

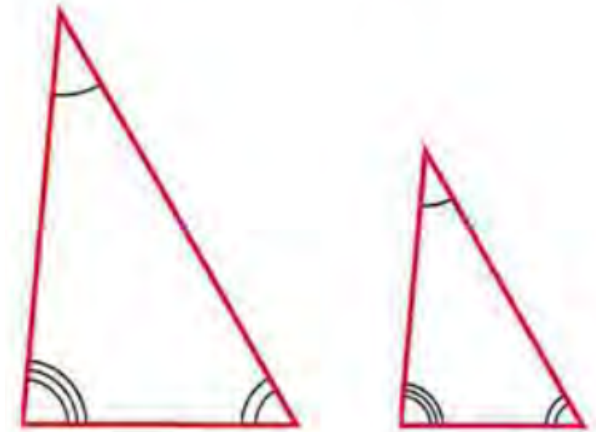
$80^\circ$



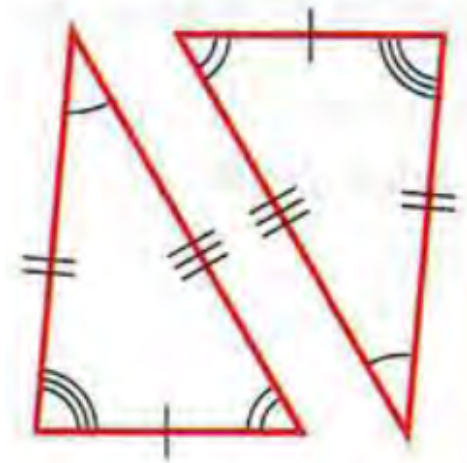
# 1.2 - Similar Triangles

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**Similar triangles** -  
corresponding angle measures  
equal and side measures  
proportional



**Congruent triangles** - equal  
corresponding angle measures  
and equal corresponding side  
measures



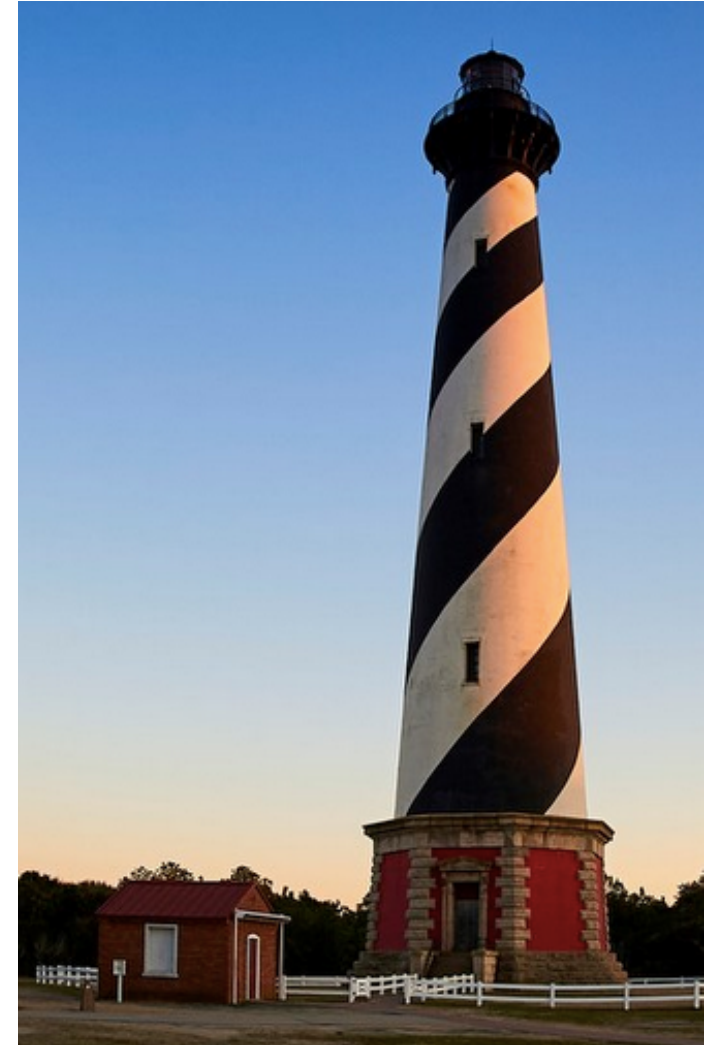


# 1.2 - Similar Triangles

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**Cape Hatteras Lighthouse** - The lighthouse is the tallest in North America. If a 5 ft woman casts a  $1\frac{1}{2}$  foot shadow and the lighthouse casts a 48 ft shadow, approximately how tall is the lighthouse?

**160 ft**



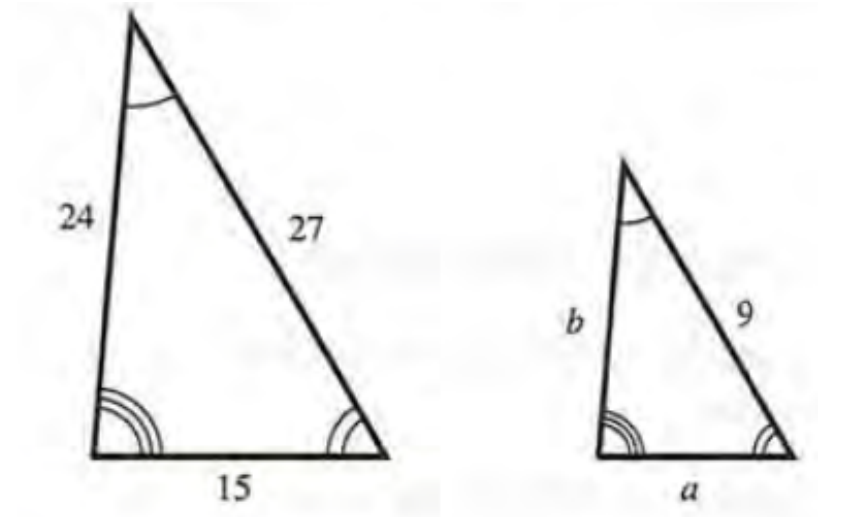
# 1.2 - Similar Triangles

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## Practice

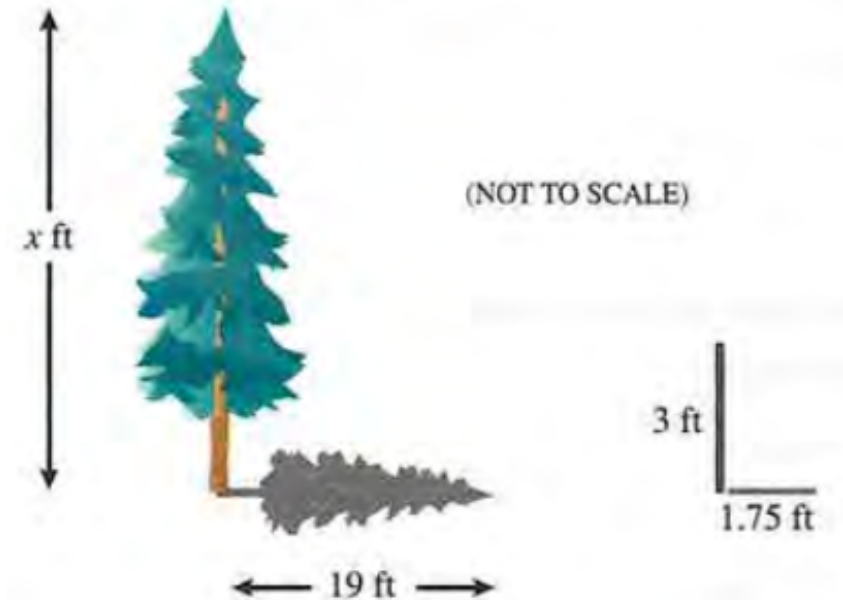
a. Find the unknown sides

$$a = 5$$
$$b = 8$$



b. Billy wants to find the height of a tree. He measures the shadow of the tree. He also measures the height and shadow of a stake in the ground.

$$x \approx 32.57 \text{ ft}$$





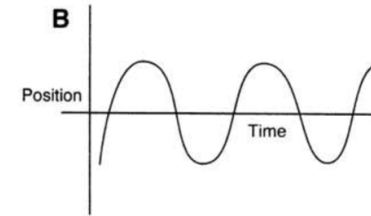
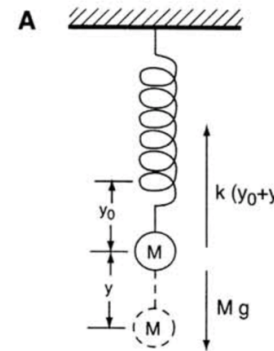
# Trigonometric Ratios

# 1.3 - Right Triangle Ratios

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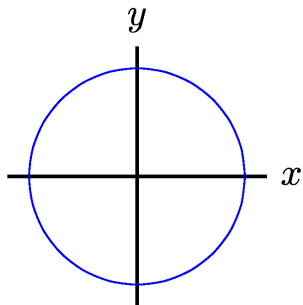
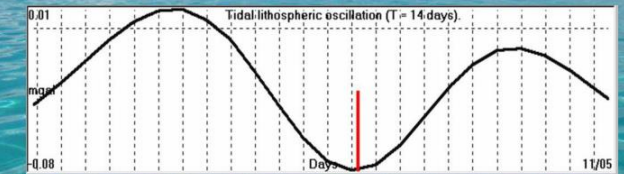
- Uses of Trigonometry

- Oscillation
- Recurring events
- Parametric equations
- Polar rose curves
- AC current
- ...



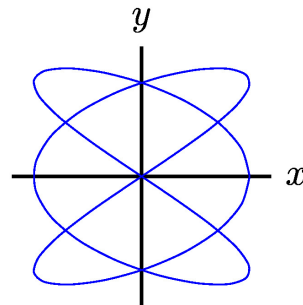
## Tidal Oscillation

- **Tidal Oscillation:** the slow, rocking motion of ocean water that occurs as the tidal bulges move around the ocean basins.



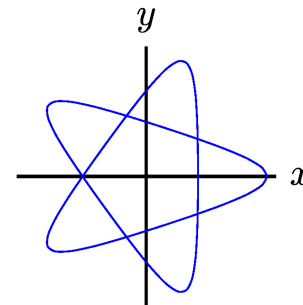
$$x(t) = \cos(t)$$

$$y(t) = \sin(t)$$



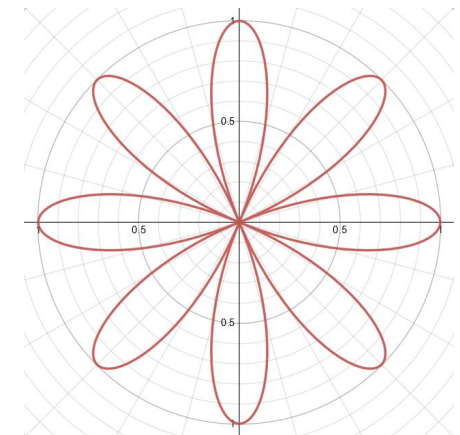
$$x(t) = 3 \cos(t)$$

$$y(t) = 2 \sin(t)$$



$$x(t) = 2 \cos(x) + 5 \cos\left(\frac{2}{3}x\right)$$

$$y(t) = 2 \sin(x) + 5 \sin\left(\frac{2}{3}x\right)$$

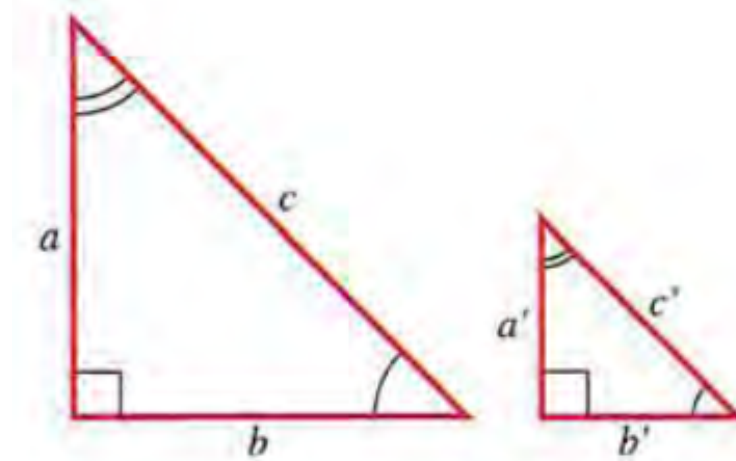


# 1.3 - Right Triangle Ratios

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Trigonometry is based on:

**Similar right triangles** - corresponding angle measures equal and side measures proportional

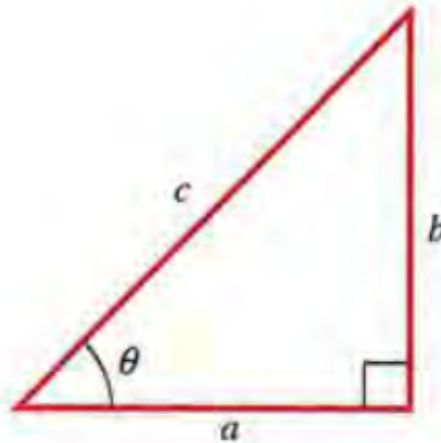


$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} \longrightarrow \frac{a}{b} = \frac{a'}{b'} \quad \text{and} \quad \dots \quad \frac{b}{c} = \frac{b'}{c'}$$

# 1.3 - Right Triangle Ratios

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**Trigonometric ratios** - based on the ratio of the sides of a right triangle and one of the *acute angles*.



$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

$$\sin \theta = \frac{b}{c}$$

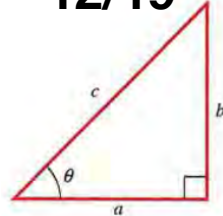
$$\cos \theta = \frac{a}{c}$$

$$\tan \theta = \frac{b}{a}$$

# 1.3 - Right Triangle Ratios

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**Main identities** - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).



$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

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## Setting Your Calculator

We are using degrees - "D" or "DEG" on calculator

Another setting is radians - "R" or "RAD" on calculator

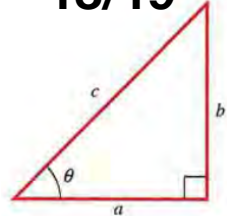
FYI,  $360^\circ = 2\pi$  radians

**DO NOT USE!!! (yet)**

# 1.3 - Right Triangle Ratios

13/19

**Main identities** - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).



$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

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Find the values  $x$  and  $y$  to the nearest hundredth.

$$\sin 67 = \frac{x}{120}$$

$$x = 120 \sin 67$$

$$x \approx 120(0.9205)$$

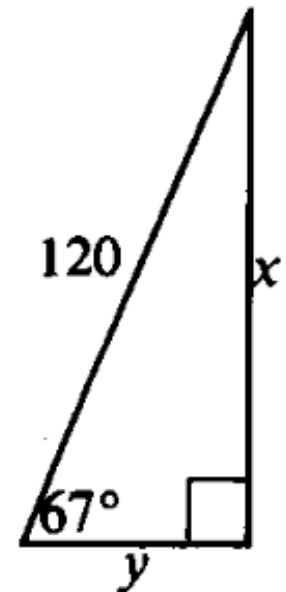
$$x \approx \mathbf{110.46}$$

$$\cos 67 = \frac{y}{120}$$

$$y = 120 \cos 67$$

$$y \approx 120(0.3907)$$

$$y \approx \mathbf{46.89}$$

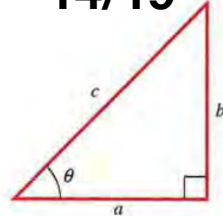




# 1.3 - Right Triangle Ratios

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**Main identities** - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).

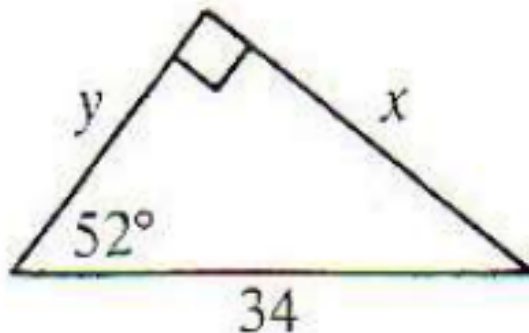


$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

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**Practice:** Find the values of  $x$  and  $y$  to the nearest hundredth.

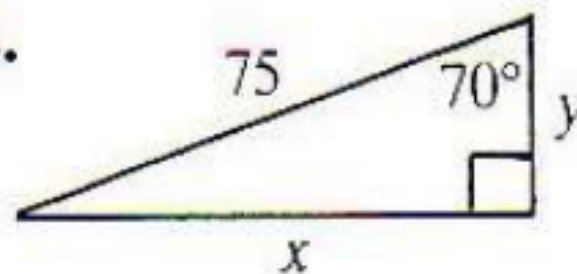
11.



$$x \approx 26.79$$

$$y \approx 20.93$$

12.



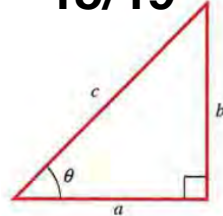
$$x \approx 70.48$$

$$y \approx 25.65$$

# 1.3 - Right Triangle Ratios

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**Main identities** - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).



$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

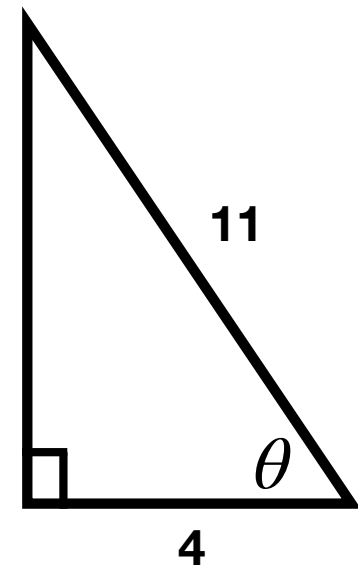
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What about finding  $\theta$ ?

$$\cos \theta = \frac{4}{11}$$

$$\cos^{-1}(\cos \theta) = \cos^{-1}\left(\frac{4}{11}\right)$$

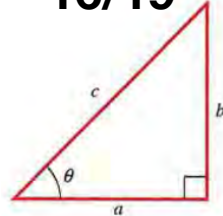
$$\theta \approx 68.68^\circ$$



# 1.3 - Right Triangle Ratios

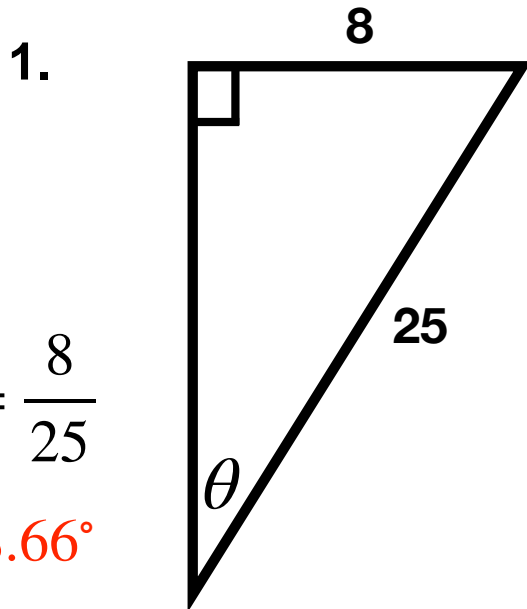
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**Main identities** - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).



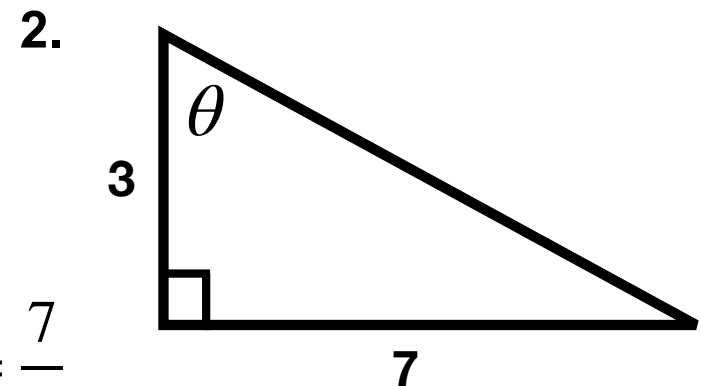
$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

**Practice:** Find  $\theta$ .



$$\sin \theta = \frac{8}{25}$$

$$\theta \approx 18.66^\circ$$



$$\tan \theta = \frac{7}{3}$$

$$\theta \approx 66.8^\circ$$

# 1.3 - Right Triangle Ratios

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**Main identities** - the three main trigonometric identities (Mnemonic: SOH-CAH-TOA).

$$\sin \theta = \frac{\textit{opposite}}{\textit{hypotenuse}} \quad \cos \theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} \quad \tan \theta = \frac{\textit{opposite}}{\textit{adjacent}}$$

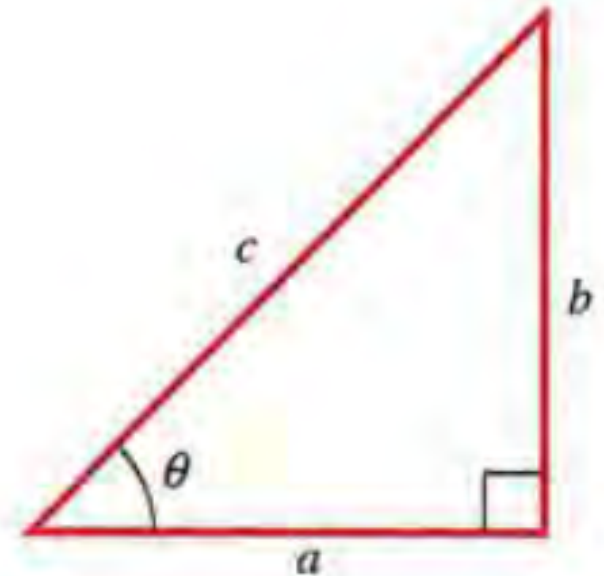
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Cosecant, secant, and cotangent are reciprocals of ...

$$\csc \theta = \frac{1}{\sin \theta} = \frac{c}{b}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{c}{a}$$

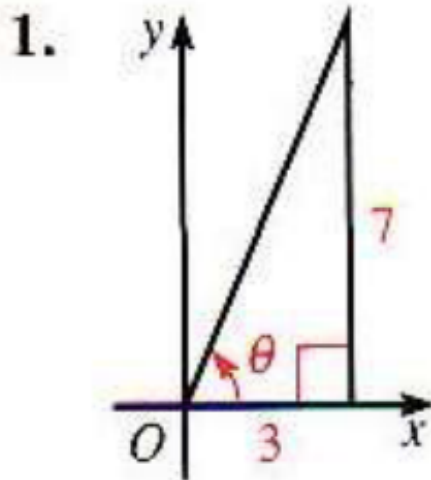
$$\cot \theta = \frac{1}{\tan \theta} = \frac{a}{b}$$



# 1.3 - Right Triangle Ratios

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**Practice:** Give the values of the six trigonometric functions of  $\theta$ .



$$\sin \theta = \frac{7\sqrt{58}}{58}$$

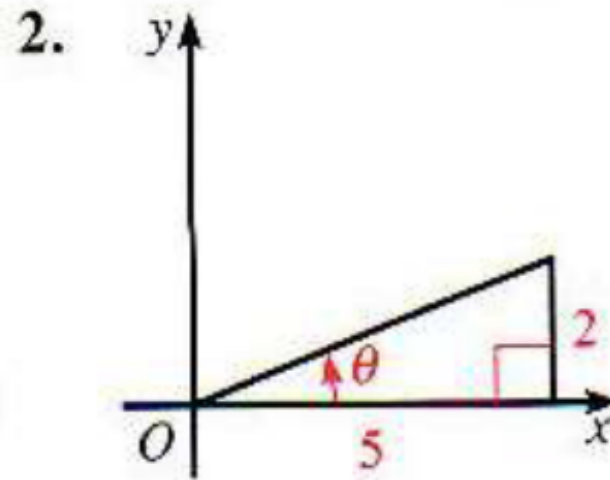
$$\csc \theta = \frac{\sqrt{58}}{7}$$

$$\cos \theta = \frac{3\sqrt{58}}{58}$$

$$\sec \theta = \frac{\sqrt{58}}{3}$$

$$\tan \theta = \frac{7}{3}$$

$$\cot \theta = \frac{3}{7}$$



$$\sin \theta = \frac{2\sqrt{29}}{29}$$

$$\csc \theta = \frac{\sqrt{29}}{2}$$

$$\cos \theta = \frac{5\sqrt{29}}{29}$$

$$\sec \theta = \frac{\sqrt{29}}{5}$$

$$\tan \theta = \frac{2}{5}$$

$$\cot \theta = \frac{5}{2}$$

# 1.3 - Right Triangle Ratios

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**Practice:** Ishaan and Sania are planning to hike to the top of a hill. If  $\theta$  is the angle formed by the hill and the ground such that  $\sec \theta = 1.75$ , find  $\sin \theta$ .

$$\sin \theta = 0.821$$

